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Book Review

Diethard Pallaschke and Stefan Rolewicz, *Foundations of Mathematical Optimization: Convex Analysis without Linearity* [Mathematics and Its Applications, Vol. 388]. Kluwer Academic Publishers, Dordrecht/Boston/London, 1997, 596 pages, ISBN 0-7923-4424-3, US$ 254.00.

Convex analysis is fundamental to the understanding and solution of mathematical optimization problems. It is well known that in convex analysis, linearity plays a key role. For example, using the calculus of linear spaces, linear supports, linear functions and linear approximations, we can develop the notions of convex sets and convex functions and explore their basic properties, including those of theoretical and practical uses in mathematical optimization. Among the properties in optimization that can be developed using convex analysis with linearity are properties concerning the existence of optimal solutions, necessary and sufficient conditions for optimality, optimality and stability under perturbations, and so on.

The goal of this book is to develop a convex analysis *without* linearity and to apply it to the field of mathematical optimization. It is hoped that the results in this book will be useful in the application of non-standard methods to optimization problems.

The book consists of 10 chapters. In the first three chapters, the fundamentals of convex analysis without linearity are proposed. In Chapter 1, this convex analysis is developed over a set called a space that does not have any type of topology. To do so, only an order structure is used. Notions of subgradients, convex sets, convex functions, duality and optimization problems under these conditions are developed. In Chapter 2, optimization problems in metric spaces are considered. Generalizations of the Asplund results about Fréchet differentiability of convex functions on a dense subset and of duality between local uniform convexity and local uniform smoothness, for example, are thereby obtained. In Chapter 3, properties of set-valued functions are investigated. The motivation is to obtain some of the tools needed to develop the theory of marginal functions in optimization.

In Chapter 4, the minimization of linear functionals on convex sets is explored. Well-posed and weakly well-posed problems in Banach spaces are considered. The properties developed in this chapter have many applications, including applications to convergence analysis for optimization problems, for instance. In Chapter 5, duality, Fenchel conjugate functions, the density of the points of differentiability of a convex function, and some necessary conditions for optimality are presented.

Chapters 6 and 7 develop necessary and sufficient optimality conditions of first order and, in the presence of equality and inequality constraints, of higher order. To develop the sufficient conditions for optimality of higher order, a method of reduction of constraints is used.

Necessary and sufficient conditions for optimality in the presence of nondifferentiable functions are developed in Chapter 8. The notion of difference convex (d.c.) functions plays an important role in this development. In Chapter 9, algorithms for solving various optimization problems in Euclidean space are given. The problems include the nonlinear complementarity problem, the continuous set covering problem, and the problem of solving the Karush–Kuhn–Tucker system. In Chapter 10, the field of vector optimization is investigated, including the notions of efficiency, proper efficiency and their relationships to scalar problems. In addition, the domination property of vector optimization is examined.

This valuable book makes a significant, new and substantial contribution to the fields of analysis and mathematical optimization. The mathematics is particularly well done, and the coverage of topics is quite broad. The prerequisites for reading this book are knowledge of functional analysis, topology and measure theory. I recommend this book without reservations to graduate students and researchers in mathematics, mathematical programming, engineering, economics, operations research, and the sciences who possess the necessary mathematical prerequisites for reading the book.

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